

# Supersymmetry breaking in 5-dimensional space-time with $S^1/Z_2$ compactification

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## Abstract

We consider supersymmetric models in 5-dimensional space-time compactified on  $S^1/Z_2$  orbifold where  $N = 2$  supersymmetry is explicitly broken down to  $N = 1$  by the orbifold projection. We find that the residual  $N = 1$  supersymmetry is broken spontaneously by a stable classical wall-like field configurations which can appear even in the simple models discussed. We also consider some simple models of bulk fields interacting with those localized on the 4-dimensional boundary wall where  $N = 1$  supersymmetry can survive in a rather non-trivial way.

# 1 Introduction

The remarkable success in the understanding of non-perturbative aspects of string theories gives a new insights into the particle phenomenology. One of the phenomenologically most promising approach has been proposed by Hořava and Witten within the 11-dimensional supergravity compactified on  $S^1/Z_2$  orbifold that is, on an interval of the length  $R$  bounded by mirror hyperplanes [1]. This theory gives the strongly-coupled limit of the heterotic string theory with two sets of  $E_8$  super-Yang-Mills theories residing on each of the two 10-dimensional hyperplanes of the orbifold and the supergravity fields living in the full 11-dimensional bulk. The important property of this model is that when  $R$  is increased, the 11-dimensional Planck mass decreases as  $R^{-1}$ , while the  $E_8$  gauge coupling remains fixed. This allows to achieve unification of gauge and gravitational couplings at a grand unification scale  $\simeq 10^{16}$  GeV [2] inferred in turn from the low-energy values of gauge couplings which are measured with very higher accuracy at Z-peak. Further, the above construction has initiated even more dramatic reduction of the fundamental higher-dimensional Planck mass down to the TeV scale with a millimeter size extra dimensions [3] and the models with TeV scale unification [4].

Obviously, in order to get a realistic phenomenology one has to compactify 6 of the 10 remaining dimensions, transverse to  $R$  on a 6-dimensional volume. After such a compactification one obtains 5-dimensional theory on an interval with mirror-plane boundaries [5] which can be described as a 5-dimensional supergravity field theory, with some possible additional bulk supermultiplets, coupled to matter superfields residing on the boundary walls. If  $R$  is the largest dimension in this set-up then one can ignore the finite volume of the 6-dimensional compactified space. The minimal supersymmetric Standard model (MSSM) in such a field theoretical limit has been constructed in [6] and we below closely follow to this construction.

One of the most important issue of the MSSM phenomenology is the mechanism of supersymmetry breaking and the origin of soft masses. A commonly accepted scenario is to break supersymmetry either spontaneously or dynamically in the hidden sector. This breaking then shows up in the visible sector due to either gauge or gravitational interactions. In the Hořava-Witten theory compactification matter could be at a strong coupling regime on one boundary, and could break supersymmetry on this boundary dynamically [1] through the gaugino condensation. Then the supersymmetry breaking effects can be transmitted to the other boundary by gravitational [7] or gauge [8] fields propagating in the 11- (or 5) dimensional bulk. More recently it was also shown [9] that supersymmetry breaking from the one boundary to another can be mediated through the super-Weyl anomaly [9]. Thus, in these models, fields living on one of the boundaries play the role of the hidden sector for the fields living on another boundary.

A distinct higher-dimensional source of supersymmetry breaking is provided by the Scherk-Schwarz mechanism [10] where non-trivial boundary conditions for the fields along the compactified dimensions are responsible for the supersymmetry breaking. This mechanism has been studied recently within the framework of large extra dimensions as well [11].

In the present paper we investigate the possibility of the breaking of a rigid supersymmetry in 5-dimensional field-theoretic limit of the Hořava-Witten compactification [1]. We will show

that there could be a new source of supersymmetry breaking that relied on the Dvali-Shifman mechanism of supersymmetry breaking [12]. Particularly, we will argue that in a wide class of models with bulk supermultiplets under a certain boundary condition imposed there appear classical stable wall-like field configurations that break the residual  $N = 1$  supersymmetry spontaneously, while the initial  $N = 2$  supersymmetry is explicitly broken down to  $N = 1$  due to the orbifold projection. We will also give some simple examples where  $N = 1$  supersymmetry can survive in a rather non-trivial way.

## 2 Supersymmetry in 5 dimensions compactified on $S^1/Z_2$

In this section we introduce various supersymmetric multiplets in 5-dimensional space-time subject to the  $S^1/Z_2$  compactification.  $N = 1$  supersymmetric 5-dimensional multiplets can be easily deduced from the  $N = 2$  4-dimensional ones (see e.g. [13]). Throughout the paper capitalized indices  $M, N = 0, \dots, 4$  will run over 5-dimensional space-time, while those of lower-case  $m, n = 0, \dots, 3$  will run over its 4-dimensional subspace;  $i = 1, 2$  and  $a = 1, 2, 3$  will denote  $SU(2)$  spinor and vector indices, respectively. We work with metric with the most negative signature  $\eta_{MN} = \text{diag}(1, -1, -1, -1, -1)$  and take the following basis for the Dirac matrices:

$$\Gamma^M = \left( \left( \begin{array}{cc} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{array} \right), \left( \begin{array}{cc} -iI_2 & 0 \\ 0 & iI_2 \end{array} \right) \right), \quad (1)$$

where  $\sigma^m = (I_2, \vec{\sigma}) = \bar{\sigma}_m$  ( $I_2$  is a  $2 \times 2$  identity matrix) and  $\vec{\sigma}(\sigma^1, \sigma^2, \sigma^3)$  are the standard Pauli matrices. Symplectic-Majorana spinor is defined as a  $SU(2)$ -doublet Dirac spinor  $\chi^i$  subject to the following constraints:

$$\chi^i = c^{ij} C \bar{\chi}^{jT}, \quad (2)$$

where

$$c = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } C = \begin{pmatrix} c & \\ & c \end{pmatrix} \quad (3)$$

are  $2 \times 2$  and  $4 \times 4$  charge conjugation matrices, respectively. A symplectic-Majorana spinor (2) can be decompose into the 4-dimensional chiral fermions as:

$$\chi^i = \begin{pmatrix} \chi_L^i \\ \chi_R^i \end{pmatrix}, \quad (4)$$

where two-component chiral fermions  $\chi_{L,R}^i$  are related to each other according to equation

$$\chi_{L(R)}^i = c^{ij} c \bar{\chi}_{R(L)}^{j*} \quad (5)$$

**Hypermultiplet.** The 5-dimensional off shell hypermultiplet  $\mathcal{H} = (h^i, \psi, F^i)$  consist the scalar field  $h^i$  ( $i = 1, 2$ ) being a doublet of  $SU(2)$ , an  $SU(2)$ -singlet Dirac fermion  $\psi = (\psi_L, \psi_R)^T$  and  $SU(2)$ -doublet  $F^i$ , being an auxiliary field These fields form two  $N = 1$  4-dimensional chiral

multiplets  $H_1 = (h^1, \psi_L, F^1)$  and  $H_2 = (h^2, \psi_R, F^2)$ . The supersymmetry transformation laws are:

$$\begin{aligned}\delta_\xi h^i &= \sqrt{2} c^{ij} \bar{\xi}^j \psi, \\ \delta_\xi \psi &= -i\sqrt{2} \Gamma^M \partial_M h^i c^{ij} \xi^j - \sqrt{2} F^i \xi^i, \\ \delta_\xi F^i &= i\sqrt{2} \bar{\xi}^i \Gamma^M \partial_M \psi\end{aligned}\tag{6}$$

while the corresponding 5-dimensional Lagrangian has the form:

$$\mathcal{L}_{hyper}^{(5)} = \left(\partial_M h^i\right)^+ \left(\partial^M h^i\right) + i\bar{\psi} \Gamma^M \partial_M \psi + \left(F^i\right)^+ \left(F^i\right)\tag{7}$$

In order to project the above structure down to a 4-dimensional  $N = 1$  supersymmetric theory on the boundary wall one should define the transformation properties of fields entering in the hypermultiplet under the discrete  $Z_2$  orbifold symmetry. The  $Z_2$  acts on the fifth coordinate as  $x^4 \rightarrow -x^4$ . A generic bosonic field  $\varphi(x_m, x_4)$  transforms like

$$\varphi(x^m, x^4) = \mathcal{P} \varphi(x^m, -x^4)\tag{8}$$

while the fermionic one  $\eta(x^m, x^4)$  transforms as:

$$\eta(x^m, x^4) = \mathcal{P} i \sigma^3 \Gamma^4 \eta(x^m, -x^4),\tag{9}$$

where  $\mathcal{P}$  is an intrinsic parity equal to  $\pm 1$ . One can assignee the eigenvalues of the parity operator to the fields considered as in Table 1. so the bulk Lagrangian is invariant under the action of  $\mathcal{P}$ . Then on the wall located at  $x_4 = 0$  the transformations (6) are reduced to the following  $N = 1$  supersymmetry transformations of the even-parity states generated by parameter  $\xi_L^1$ :

$$\begin{aligned}\delta_\xi h^1 &= \sqrt{2} \xi_L^{1T} c \psi_L, \\ \delta_\xi \psi_L &= i\sqrt{2} \sigma^m c \xi_L^{1*} \partial_m h^1 - \sqrt{2} \xi_L^1 \left(F^1 + \partial_4 h^2\right), \\ \delta_\xi \left(F^1 + \partial_4 h^2\right) &= i\sqrt{2} \xi_L^{1+} \bar{\sigma}^m \partial_m \psi_L\end{aligned}\tag{10}$$

being the usual  $N = 1$  supersymmetry transformations for the chiral multiplet. Thus what we have on the boundary wall is the simplest non-interacting massless Wess-Zumino model. Note, that an effective auxiliary field for the chiral multiplet contains the derivative term  $\partial_4 h^2$  which is actually even under the  $Z_2$  orbifold transformation. Thus the expectation value of  $\partial_4 h^2$  plays the role of the order parameter of supersymmetry breaking on the boundary wall.

**Vector supermultiplet.** Now let us consider a 5-dimensional  $SU(N)$  Yang-Mills supermultiplet  $\mathcal{V} = (A^M, \lambda^i, \Sigma, X^a)$ . It contains a vector field  $A^M = A^{M\alpha} T^\alpha$ , a real scalar  $\Sigma = \Sigma^\alpha T^\alpha$ , an  $SU(2)$ -doublet gaugino  $\lambda^i = \lambda^{i\alpha} T^\alpha$  and an  $SU(2)$ -triplet auxiliary field  $X^a = X^{a\alpha} T^\alpha$  all in the adjoint representation of the gauge  $SU(N)$  group. Here  $\alpha = 1, \dots, N$  runs over the  $SU(N)$  indices

Parity, $\mathcal{P}$	+	+	+	-	-	-	-
Hypermultiplet	$h^1$	$\psi_L$	$F^1$		$h^2$	$\psi_R$	$F^2$
Vector multiplet	$A_m$	$\lambda_L^1$	$X^3$	$A_5$	$\Sigma$	$\lambda_L^2$	$X^{1,2}$

Table 1: An intrinsic parity  $\mathcal{P}$  of various fields. We define the supersymmetry transformation parameter  $\xi_L^1$  to be even ( $\mathcal{P} = 1$ ), while the parameter  $\xi_L^2$  to be odd ( $\mathcal{P} = -1$ )

and  $T^\alpha$  are the generators of  $SU(N)$  algebra  $[T^\alpha, T^\beta] = if^{\alpha\beta\gamma}T^\gamma$  with  $Tr[T^\alpha, T^\beta] = \frac{1}{2}\delta^{\alpha\beta}$ . This  $N = 2$  supermultiplet consists of an  $N = 1$  4-dimensional vector  $V = (A^m, \lambda_L^1, X^3)$  and a chiral supermultiplets  $\Phi = (\Sigma + iA^4, \lambda_L^2, X^1 + iX^2)$ . Under the  $N = 2$  supersymmetry transformations the fields of the vector supermultiplet  $\mathcal{V}$  transform as:

$$\begin{aligned}
\delta_\xi A^M &= i\bar{\xi}^i \Gamma^M \lambda^i, \\
\delta_\xi \Sigma &= i\bar{\xi}^i \lambda^i, \\
\delta_\xi \lambda^i &= \left( \sigma^{MN} F_{MN} - \Gamma^M D_M \Sigma \right) \xi^i - i(X^a \sigma^a)^{ij} \xi^j, \\
\delta_\xi X^a &= \bar{\xi}^i (\sigma^a)^{ij} \Gamma^M D_M \lambda^j - i \left[ \Sigma, \bar{\xi}^i (\sigma^a)^{ij} \lambda^j \right],
\end{aligned} \tag{11}$$

where once again the symplectic Majorana spinor  $\xi^i$  is the parameter of supersymmetric transformations,  $D_M$  is the usual covariant derivative,  $D_M \Sigma(\lambda^i) = \partial_M \Sigma(\lambda^i) - i[A_M, \Sigma(\lambda^i)]$ , and  $\sigma^{MN} = \frac{1}{4}[\Gamma^M, \Gamma^N]$ .

As in the case of the hypermultiplet we define an intrinsic parity  $\mathcal{P}$  for the fields in  $\mathcal{V}$  (see Table 1), thus projecting it down to a 4-dimensional  $N = 1$  supersymmetric vector multiplet residing on the orbifold boundary. Let  $\xi_L^1$  be the supersymmetry parameter of  $N = 1$  supersymmetric transformations on the boundary. Then on the boundary at  $x_4 = 0$ , the supersymmetric transformations (11) for the even-parity ( $\mathcal{P} = 1$ ) states reduces to:

$$\begin{aligned}
\delta_\xi A^m &= i\xi_L^{1+} \bar{\sigma}^m \lambda_L^1 + h.c., \\
\delta_\xi \lambda_L^1 &= \sigma^{mn} F_{mn} \xi_L^1 - i(X^3 - \partial_4 \Sigma) \xi_L^1, \\
\delta_\xi (X^3 - \partial_4 \Sigma) &= \xi_L^{1+} \bar{\sigma}^m D_m \lambda_L^1 + h.c.
\end{aligned} \tag{12}$$

These are indeed the transformation laws for an  $N = 1$  4-dimensional vector multiplet  $V = (A^m, \lambda_L^1, D)$  with an auxiliary field  $D = X^3 - \partial_4 \Sigma$  [8]. Once again the derivative term  $\partial_4 \Sigma$  enters into the effective auxiliary field on the boundary. Finally, the bulk Lagrangian invariant under the above supersymmetric transformations looks as:

$$\begin{aligned}
\mathcal{L}_{Yang-Mills}^{(5)} &= -\frac{1}{2g_5^2} Tr(F_{MN})^2 \\
&+ \frac{1}{g_5^2} \left( Tr(D_M \Sigma)^2 + Tr(\bar{\lambda} i \Gamma^M D_M \lambda) + Tr(X^a)^2 - Tr(\bar{\lambda} [\Sigma, \lambda]) \right)
\end{aligned} \tag{13}$$

The explicit breaking of  $N = 2$  supersymmetry down to the  $N = 1$  by the orbifold projection discussed in this section is rather transparent from analyzing general  $N = 2$  supersymmetry algebra

$$\{Q^i, Q^j\} = c^{ij} \Gamma^M C P_M + C c^{ij} Z, \tag{14}$$

where  $Z$  is a central charge. The relation (14) is invariant under the  $Z_2$  orbifold transformations:

$$\begin{aligned} Q^i &\longrightarrow i(\sigma^3)^{ij} \Gamma^4 Q^j, \\ Z &\longrightarrow -Z, \\ P_m &\longrightarrow P_m, \quad P_4 \longrightarrow -P_4. \end{aligned} \tag{15}$$

Then modding out the  $Z_2$  orbifold symmetry

$$\begin{aligned} Q_R^1 &= Q_L^2 = 0, \\ Z &= 0, \quad P_4 = 0 \end{aligned} \tag{16}$$

we obtain from (14):

$$\{Q^i, Q^j\} = c^{ij} \Gamma^m C P_m, \tag{17}$$

Taking now into account that supercharges should satisfy chirality condition given by (16) and equation (5) we finally arrive to the familiar  $N = 1$  supersymmetric algebra in 4 dimensions:

$$\{Q^1, Q^1\} = \Gamma^m C P_m. \tag{18}$$

Note however, that since (14) is operatorial equation the right hand side of (17) can be modified by the terms  $P_4 \pm Z$  when acting on the parity odd state as it was the case for the models considered in this section.

### 3 Supersymmetry breaking

**Free fields in the bulk.** We begin our discussion of supersymmetry breaking from the simplest supersymmetric models considered in the previous section. The models similar to those described above are often used to construct phenomenologically viable theories, such as MSSM on the 4-dimensional boundary [6]. The remaining  $N = 1$  supersymmetry on the boundary can be broken through the Scherk-Schwarz mechanism [10] by requiring that MSSM superpartners satisfy non-trivial boundary conditions which in turn result in the soft-breaking masses [6],[11].

However, there could exist a distinct source of supersymmetry breaking that relied on the Dvali-Shifman mechanism [12]. This mechanism is based on the fact that any field configuration which is not BPS state and breaks translational invariance breaks supersymmetry totally as well. Such a stable non-BPS configurations with a purely finite gradient energy can also appear in a compact spaces (or, more generally, in spaces with a finite volume) if there exist moduli forming a continuous manifold of supersymmetric states. This is indeed the case in the 5-dimensional models considered in the previous section.

To be more specific consider first the case of the pure hypermultiplet in the 5-dimensional bulk. The corresponding Lagrangian (11) describes a system of free massless fields. Thus, it seems that any constant value of these fields will be a ground state of the model and, moreover,

these ground states will preserve supersymmetry. However this is not the case for the parity-odd fields, in particular for the complex scalar field  $h^2$ , since  $\langle h^2 \rangle = \text{const}$  does contradict the boundary condition given by (8) with  $\mathcal{P}(h^2) = -1$ . Thus the boundary condition singles out the trivial configuration  $\langle h^2 \rangle = 0$  among all constant states. Besides this trivial configuration however, there could actually be stable non-trivial configurations as well. One of them is the constant phase configuration that linearly depends on the fifth coordinate:

$$\langle h^2 \rangle = \epsilon x^4, \quad (19)$$

where  $\epsilon$  is an arbitrary constant which can be chosen to be real. The configuration (19) is odd under the  $Z_2$  orbifold transformation as it should be and obviously breaks translational invariance in  $x_4$  direction. However, the configuration (19) does not satisfy the ordinary periodicity condition on a  $S^1$  circle,  $h^2(x^m, x^4 + 2R) = h^2(x^m, x^4)$ , but rather the modified one, which we define as:

$$h^2(x^m, x^4 + 2R) = h^2(x^m, x^4) + 2\epsilon R, \quad (20)$$

Then the configuration  $\langle h^2 \rangle$  interpolates from 0 to  $2\epsilon R$  when one makes a full circle around the compactified dimension<sup>1</sup>. Clearly, the Lagrangian density  $\mathcal{L}_{hyper}^{(5)}$  (7) remains single-valued and periodic,  $\mathcal{L}_{hyper}^{(5)}(x^4 + 2R) = \mathcal{L}_{hyper}^{(5)}(x^4)$ , so the theory with boundary condition (20) will be consistent as it was in the case of the ordinary periodic boundary conditions. Thus, if we assume that  $h^2$  and its superpartner  $\psi_R$  are defined modulo  $2\epsilon R$  on  $S^1/Z_2$  space, then the configuration (19) will be perfectly compatible with  $S^1/Z_2$  orbifold symmetries.

Stability of the above configuration (19) can be straightforwardly checked by performing finite deformation  $\langle h^2 \rangle + \delta h^2$ , where  $\delta h^2 \rightarrow 0$  at infinity. Then the variation of an energy functional:

$$E = \int d^5x \left[ \left( \partial_m h^2 \right)^2 + \left| \frac{\partial h^2}{\partial x^4} \right|^2 \right] \quad (21)$$

is indeed zero for the configuration (19):

$$\frac{\delta E}{\delta h^{2*}} = -\partial_4 \partial^4 h^2 = 0 \quad (22)$$

Being stable, we can treat the configuration (19) as a possible vacuum state of the model. While (19) is multiply defined, the vacuum energy density is a constant given by the purely gradient energy  $\mathcal{E} = \epsilon^2$ . One can see that to see that this vacuum configuration spontaneously breaks remaining  $N = 1$  supersymmetry on the boundary wall. Indeed, the effective  $F$ -term on the boundary (see (10)) is non-zero,  $\langle F^1 + \partial_4 h^2 \rangle = \epsilon \neq 0$ , indicating the spontaneous breaking of  $N = 1$  supersymmetry.

Despite of the fact that the supersymmetry is completely broken all fields in the model remain massless, so it looks such as the Fermi-Bose degeneracy is still present. The reason for

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<sup>1</sup>Similar but BPS configurations on a non - simply connected compact spaces in lower dimensions have been considered in [14] (see also [15]) and further explored in [16]. Contrary, non - BPS and thus supersymmetry breaking configurations in models with twisted boundary conditions are discussed in [17]

such a degeneracy is following. All four real components of  $h^2$  and  $h^1$  are massless, because all of them are the goldstone bosons: one mode corresponds to the spontaneously broken translational invariance and another three correspond to the complete spontaneous breaking of the global  $SU(2)$  invariance. The massless fermion  $\psi_L$  is a goldstino of the spontaneously broken  $N = 1$  supersymmetry on the boundary wall. Obviously, if one gauges the model, all these massless states will give rise to the masses of the corresponding gauge fields (graviphoton, gravitino and  $SU(2)$  gauge fields) through the (super)Higgs mechanism.

Now let us turn to the  $Z_2$ -even scalar field  $h^1$  from the  $H_1 = (h^1, \psi_L, F^1)$  chiral supermultiplet. It is obvious, that all  $x^4$ -dependent configurations of  $h^1$  will be unstable and only those being the trivial constant can be realized as a vacuum states. These vacuum states are supersymmetry preserving. Beside the trivial homogenous configurations, however, there can exist also  $x^4$ -independent stable configuration with a winding phase:

$$\langle h^1 \rangle = \epsilon r \sin \theta e^{i\varphi}, \quad (23)$$

where  $r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$  and  $\varphi$  and  $\theta$  are azimuthal and zenith angles in  $\{x^1, x^2, x^3\}$  plane. The configuration (23) is indeed a solution of the equation of motion:

$$\begin{aligned} & -\partial_M \partial^M h^1 \equiv \\ & \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial h^1}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial h^1}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left( \frac{\partial^2 h^1}{\partial \varphi^2} \right) + \frac{\partial^2 h^1}{\partial (x^4)^2} - \frac{\partial^2 h^1}{\partial (x^0)^2} = 0 \end{aligned} \quad (24)$$

and compatible with  $S^1/Z_2$  orbifold symmetries.

The solution (23) for any hyperplane  $\theta = \text{const} \neq \pi n$  reminds the ordinary global cosmic string configuration, except that the modulus of the scalar field  $h^1$  never assumes a constant value. Obviously, this configuration also breaks supersymmetry. Actually, there can be the cases where both configurations (19) and (23) are present simultaneously.

The solution similar to the  $x^4$ -dependent constant phase configuration (19) can be obtained as well for the  $Z_2$ -odd scalar field  $\Sigma + iA_5$  as well when the case when vector supermultiplet lives in the bulk and the non-trivial boundary condition similar to (20) for the  $\Sigma + iA_5$  is assumed. This is the case not only for the Abelian (non-interacting) vector supermultiplet but also for the non-Abelian Yang-Mills supermultiplet. For the later case the interaction terms also do not contribute to the vacuum energy, and this is indeed satisfied for the vacuum configuration where all fields except of  $\Sigma$  have a zero vacuum expectation value.

Before proceeding further, let us make a comment on the case of massive non-interacting hypermultiplet. Namely if ordinary periodic boundary conditions are assumed, one can add a mass term to the Lagrangian  $\mathcal{L}_{hyper}^{(5)}$  (7):

$$\mathcal{L}_{mass}^{(5)} = m \left[ \bar{\psi} \psi + h^i c^{ij} (F^j)^+ + (h^i)^+ c^{ij} F^j \right] \quad (25)$$

since it transforms as a full derivative under the supersymmetric transformations. The condition of vanishing  $F$ -terms leads to  $\langle h^i \rangle = 0$ . So we have no more continuous manifold of degenerate



supersymmetric states the presence of which was so crucial in the massless case discussed above. However, in the case of  $S^1/Z_2$  compactification  $\mathcal{L}_{mass}^{(5)}$  is  $Z_2$ -odd, while  $\mathcal{L}_{hyper}^{(5)}$  is  $Z_2$ -even. Thus  $Z_2$ -parity actually forbids mass term  $\mathcal{L}_{mass}^{(5)}$  (25). Note, that even in the case of compactification on a simple circle  $S^1$  non-trivial boundary condition (20) forbids the existence of the mass term as well, since it is multiply-defined in this case. To conclude, the stable spatially extended configurations (19) and/or (24) most likely appear in models with free bulk superfields and if so they inevitably break supersymmetry completely.

**Interacting bulk fields.** Let us briefly discuss the theories with non-trivial interactions in the bulk in connection with the supersymmetry breaking mechanism. From the above discussion we conclude that the existence of a moduli forming a continuous manifold of degenerate supersymmetric states is a necessary ingredient for the considered supersymmetry breaking mechanism to work. Thus, one can ask: can some non-trivial interactions which presumably appear in realistic theories remove the degeneracy in the case of free supermultiplets above and in this way protect supersymmetry? Indeed, one can expect that a certain interaction removing the degeneracy can drive the classical field configurations to be supersymmetry preserving with vanishing  $F$  and  $D$  terms. It might also happen that non-vanishing potential energy ( $F$ -term) exactly cancel the gradient energy ( $D$ -term) as it is the case for the BPS configurations [14],[15],[18]. However, these models with  $N = 1$  supersymmetry in four dimensions straightforwardly applicable to the case of  $N = 2$  supersymmetry. The point is that the framework of  $N = 2$  supersymmetry is more restrictive than of  $N = 1$ , so many interactions allowed by  $N = 1$  can not be straightforwardly extended in the case of  $N = 2$ . Moreover, the orbifold symmetries seem to put further restrictions.

Interacting supersymmetric gauge theories in five dimensions have been discovered relatively recently [19]. The most general Lagrangian (with up to two derivatives and four fermions) on the Coulomb branch is

$$\mathcal{L} = \frac{1}{8\pi} \text{Im} \left[ \int d^4\theta \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} (\Phi^+ e^{2V}) + \int d^2\theta \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^2} \mathcal{W}^2 \right], \quad (26)$$

where  $\mathcal{F}(\Phi)$  is a holomorphic function. The  $N = 1$  chiral superfield  $\Phi$  along with the  $N = 1$  vector superfield  $V$  forms  $N = 2$  vector supermultiplet  $\mathcal{V}$ , and  $\mathcal{W}$  being the standard gauge field strength corresponding to  $V$ . The prepotential  $\mathcal{F}(\Phi)$  can be at most cubic [19]:

$$\mathcal{F}(\Phi) = \frac{4\pi}{g_5^2} \Phi^2 + \frac{c}{3} \Phi^3 \quad (27)$$

The first term in (27) produces just the kinetic terms in the Lagrangian (26), while the second one generates the non-trivial interaction terms. However, since the second term of the prepotential (27) is odd under the  $Z_2$  orbifold transformations one should take  $c = 0$ , to keep  $Z_2$  invariance of the Lagrangian (26) [20]. It is unlikely that this term can be generated at a quantum level (at least perturbatively) as it is the case when the fifth dimension compactifies on a simple circle [21]. Thus, we are left with theories described by the Lagrangians of type

(13) with, possibly, some additional hypermultiplets charged under the gauge group. In this case one can find the field configurations which completely breaks supersymmetry in a complete analogy to the cases that we have discussed above.

Note, however, that the Lagrangians (7) and (13) are (and therefore the total Lagrangian which includes the interaction with the bulk fields) not  $N = 2$  supersymmetric actually but rather they are  $N = 1$  supersymmetric under the constraints imposed by orbifold boundary conditions. So, if one can consider the interactions of the bulk fields with those localized on the boundary, these interactions will be explicitly  $N = 1$  supersymmetric. As we will see in the next section one can keep  $N = 1$  supersymmetry unbroken in such a cases.

## 4 Keeping supersymmetry on the boundary wall.

In this section we will consider some simple models of the bulk fields interacting with a superfields localized on the 4-dimensional boundary wall where the  $N = 1$  supersymmetry can survive. First is the model of bulk hypermultiplet interacting with the boundary  $N = 1$  chiral superfield and another is the model of  $U(1)$  gauge supermultiplet in the bulk with Fayet-Iliopoulos (FI)  $D$ -terms on the boundary. The situation that appears in these models is similar to the one discussed in [18] where the different non-compact 4-dimensional models are considered.

**Bulk fields interacting with boundary fields.** Let us consider the  $N = 1$  chiral superfield  $\Phi = (\phi, \chi_L, F_\Phi)$  localized on the 4-dimensional boundary  $x^4 = 0$ . The boundary Lagrangian has the usual form of a 4-dimensional chiral model built from an  $N = 1$  supersymmetric chiral superfields  $\Phi$ :

$$\begin{aligned} \mathcal{L}_\Phi^{(4)} = & (\partial_m \phi)^+ (\partial^m \phi) + i \chi_L^+ i \overline{\sigma}^m \partial_m \chi_L + F_\Phi^+ F_\Phi - \\ & \left[ \frac{\partial W_\Phi}{\partial \phi} F_\Phi + h.c. \right] - \frac{1}{2} \left[ \frac{\partial^2 W_\Phi}{\partial \phi \partial \phi} \chi_L^T C \chi_L + h.c. \right], \end{aligned} \quad (28)$$

where  $W_\Phi(\Phi)$  is a superpotential. Then the total Lagrangian has the form:

$$\mathcal{L}_{hyper}^{(5)} + \left[ \mathcal{L}_\Phi^{(4)} + \mathcal{L}_{\Phi H_1}^{(4)} \right] \delta(x^4), \quad (29)$$

where  $\mathcal{L}_{\Phi H_1}^{(4)}(\Phi, H_1)$  describes the interactions between the chiral superfields  $\Phi = (\phi, \chi_L, F_\Phi)$  and  $H_1 = (h^1, \psi_L, F^1 + \partial_4 h^2)$  on the boundary  $x^4 = 0$  through the superpotential  $W_{\Phi H_1}(\Phi, H_1)$ :

$$\mathcal{L}_{\Phi H_1}^{(4)} = - \left[ \frac{\partial W_{\Phi H_1}}{\partial h^1} F_{H_1} + \frac{\partial W_{\Phi H_1}}{\partial \phi} F_\Phi + h.c. \right] + \text{fermionic terms}, \quad (30)$$

where  $F_{H_1} = F^1 + \partial_4 h^2$ . The equations of motion for the  $F$ -terms resulting from the Lagrangian (29) are:

$$F^2 = 0,$$

$$\begin{aligned}
F_{\Phi}^+ &= \frac{\partial W_{\Phi}}{\partial \phi} + \frac{\partial W_{\Phi H_1}}{\partial \phi}, \\
F^{1+} &= \frac{\partial W_{\Phi H_1}}{\partial h^1} \delta(x^4),
\end{aligned} \tag{31}$$

while the equation of motion for  $\langle h^2 \rangle$  is:

$$\partial_4 \left( \partial_4 \langle h^2 \rangle + \langle F^1 \rangle \right) = 0. \tag{32}$$

Now, if  $\langle F^1 \rangle \neq 0$ , then the degeneracy in  $\langle h^2 \rangle$  is actually removed and the configuration  $\langle h^2 \rangle$  can satisfy the following equation:

$$\partial_4 \langle h^2 \rangle + \langle F^1 \rangle = 0. \tag{33}$$

For  $\langle \frac{\partial W_{\Phi H_1}}{\partial h^1} \rangle = \alpha = \text{const}$  we get from (33):

$$\langle h^2 \rangle = -\alpha \varepsilon(x^4) \equiv \begin{cases} -\alpha, & x^4 > 0 \\ \alpha, & x^4 < 0 \end{cases}. \tag{34}$$

Thus, if equation  $\langle F_{\Phi} \rangle = 0$  is satisfied additionally (that can be easily justified in general),  $N = 1$  supersymmetry remains unbroken. In the case of zero  $\langle F^1 \rangle = 0$  the degeneracy in  $\langle h^2 \rangle$  is restored and we come back to the case of free fields considered above (see eq. (19)).

**U(1) in the bulk with FI term.** Now we are going to consider the model of  $N = 1$  super-Maxwell theory in five dimensions with FI  $D$ -term. The FI term can also lift the degeneracy of supersymmetric states. If we forget for a moment about the orbifold compactification, we can straightforwardly add FI term

$$-2\eta^a X^a \tag{35}$$

to the Lagrangian (13) for the  $U(1)$  supermultiplet, where  $\eta^a$  is a  $SU(2)$ -triplet of constants. However,  $Z_2$  orbifold symmetry actually forbids such term because the auxiliary fields  $X^{1,2}$  and  $X^3$  have an opposite orbifold parity (see Table 1). The only FI term we can add in the simplest case considered here is that localized on the boundary wall

$$-2\eta \left( X^3 - \partial_4 \Sigma \right) \delta(x^4). \tag{36}$$

Then the situation becomes much similar to the case of bulk hypermultiplet interacting with boundary superfield discussed just above. Indeed, the degeneracy in  $\langle \Sigma \rangle$  is lifted for non-zero value  $\langle X^3 \rangle = g_5^2 \eta \delta(x^4)$ , since now

$$\partial_4 \langle \Sigma \rangle = \langle X^3 \rangle. \tag{37}$$

Thus the configuration  $\langle \Sigma \rangle$  is given by (34) with  $\alpha = -g_5^2 \eta$  now, and supersymmetry is indeed unbroken again.

## 5 Conclusions and outlook

The 5-dimensional supersymmetric models with  $S^1/Z_2$  orbifold compactification are often considered as a phenomenologically valuable low-energy limit of the Hořava-Witten theory. Here we considered the question of supersymmetry breaking in theories of such kind. In particular, we argue that in a wide class of semi-realistic models there typically exist the spatially extended field configurations which are not supersymmetric. On the other hand, we also considered some explicit examples of models where bulk fields interact with boundary ones and show that supersymmetry can be preserved in a rather non-trivial way.

While we have concentrated in this paper on the case of compact  $S^1/Z_2$  orbifold it to be possible to extend the main part of our discussions to the case of infinite extra dimensions with finite volume, that is for the case of the Randall-Sundrum compactification [22]. Note that  $Z_2$  orbifold symmetry also plays a crucial role for the localization of gravity on the 3-brane in the Randall-Sundrum model.

Since the Hořava-Witten theory essentially deals with supergravity it is interesting also to consider generalization of our models to the case of local supersymmetry<sup>2</sup>. Finally, more link to the realistic phenomenological models is worth to investigate. We hope to touch these issues in future publications.

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<sup>2</sup>Off-shell formulation of 5-dimensional supergravity have been recently given in [23].

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